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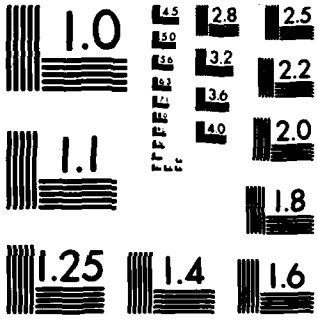

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CONFIDENCE INTERVALS USING THE REGENERATIVE METHOD FOR SIMULATION OUTPUT ANALYSIS

by

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1. INTRODUCTION

The regenerative method is a mathematically rigorous procedure for obtaining confidence intervals for steady state parameters. In order to properly assess the regenerative method, it is necessary to discuss those characteristics that make a confidence interval "good."

2. QUALITATIVE STRUCTURE OF CONFIDENCE INTERVALS

Given a parameter μ, a confidence interval for μ is generally based on a limit theorem of the form

$$(r_t - \mu)/v_t \Rightarrow L \tag{2.1}$$

as $t \to \infty$, where L is a finite r.v. with a continuous distribution function; the parameter t measures the simulation effort required to obtain r_t and v_t. The processes r_t and v_t will be called a <u>point estimate</u> (for μ) and a <u>normalizing process</u>, respectively; we shall always assume v_t is positive. To obtain an approximate $100(1 - \alpha)\%$ confidence interval for μ, select z_1, z_2 such that $P\{z_1 \leq L \leq z_2\} = 1 - \alpha$. Then, for large t,

$$[r_t - z_2 v_t,\ r_t - z_1 v_t] \tag{2.2}$$

contains μ with probability $1 - \alpha$. The following hierarchy of properties largely determines the quality of the confidence interval.

a.) <u>consistency of r_t</u>: if r_t is not consistent, v_t does not tend to zero, and confidence interval half-length does not shrink to zero with increasing t.

b.) <u>asymptotic mean square error of r_t</u>: in general, r_t is asymptotically normal. Then, there exists a non-negative σ such that

(2.3) $$t^{1/2}(r_t - \mu) \Rightarrow \sigma N(0,1) .$$

Squaring and taking expectations through (2.3), we observe that MSE $(r_t) \sim \sigma^2/t$. Consequently, one wants to choose r_t so that σ^2 is as small as possible.

c.) <u>expected half-width of confidence interval</u>: by (2.2), the expected half-width of the confidence interval is $(z_2 - z_1)Ev_t$. In general, when asymptotic normality holds, $(z_2 - z_1)Ev_t \sim (z_2 - z_1)v/t^{1/2}$ for some v ; the goal is to minimize v .

d.) <u>variability of half-width of confidence interval</u>: the variance of the half-width is given by $(z_2 - z_1)^2 \text{var } v_t$. Under quite general conditions, $(z_2 - z_1)^2 \text{var } v_t \sim (z_2 - z_1)^2\alpha/t$; the goal is to minimize α .

e.) <u>approximation error</u>: Let $\Delta_t = \left|P\{z_1 \le (r_t - \mu)/v_t \le z_2\} - P\{z_1 \le L \le z_2\}\right|$ be the coverage error for the confidence interval. Berry-Esseen considerations suggest that, in general, $\Delta_t \sim \beta/t^{1/2}$; minimization of β is desirable.

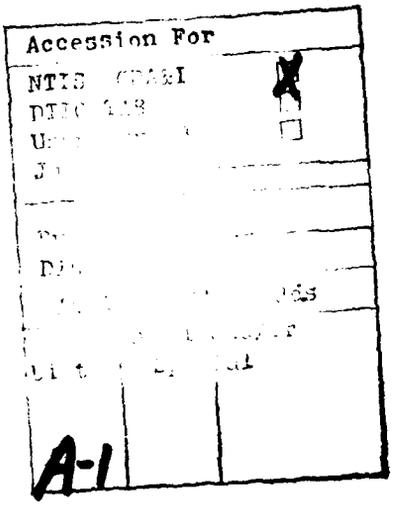
3. THE REGENERATIVE METHOD

Loosely speaking, a regenerative process is one which looks like a sequence of independent and identically distributed (i.i.d.) r.v.'s, when viewed on an appropriate random time scale. More precisely, $X = \{X(t) : t \ge 0\}$ is a regenerative process with regeneration times $0 = T_0 < T_1 < \ldots$ if $\{\tau_k, X(s) : T_{k-1} \le s < T_k\}$ is a sequence of i.i.d. random elements, where $\tau_k = T_k - T_{k-1}$. For examples of such processes, see

Crane and Lemoine (1977). Given a real-valued function defined on the state space of X,

$$r_t = \frac{1}{t}\int_0^t f(X(s))ds \to r \quad a.s. \tag{3.1}$$

under mild assumptions on X and f. The goal of a steady state simulation is to produce confidence intervals for r.



If

$$N(t) = \max\{k \geq 0 : T_k \leq t\}$$

and

$$Y_i = \int_{T_{i-1}}^{T_i} f(X(s))ds,$$

then

$$r_t \approx \bar{Y}_{N(t)}/\bar{\tau}_{N(t)} \tag{3.2}$$

where $\bar{Y}_n$, $\bar{\tau}_n$ are the sample means of the Y_i's and τ_i's, respectively. Regenerative structure ensures that $\{(Y_i, \tau_i) : i \geq 1\}$ is a sequence of i.i.d. random vectors, so that (3.1) and (3.2) together suggest that $r = EY_1/E\tau_1$. Then, by (3.2),

$$r_t - r \approx \bar{Z}_{N(t)}/\bar{\tau}_{N(t)}$$

where $Z_k = Y_k - r\,\tau_k$ has mean zero. Standard central limit theory arguments prove that

$$t^{1/2}(r_t - r) \Rightarrow \sigma N(0,1)$$

where $\sigma^2 \triangleq \sigma^2(Z_1)/E\tau_1 < \infty$, if $E(Y_1^2 + \tau_1^2) < \infty$. Furthermore, $\eta_t \to \sigma$ a.s., where $\eta_t^2 = s^2_{N(t)}/\bar{\tau}_{N(t)}$ and

$$s_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} (Y_i - (\bar{Y}_n/\bar{\tau}_n)\tau_i)^2 .$$

We conclude that

$$(r_t - r)/v_t \Rightarrow N(0,1)$$

where $v_t = |\eta_t|/t^{1/2}$ is the normalizing process for the regenerative method. The qualitative structure of the regenerative confidence interval can be summarized as follows:

a.) r_t is consistent for r,

b.) $MSE(r_t) \sim (\sigma^2(Z_1)/E\tau_1)/t$ (note that <u>any</u> confidence interval method using the sample mean r_t as a point estimate will have the same MSE),

c.) $(z_2-z_1)Ev_t \sim 2z(\alpha)\sigma(Z_1)/(E\tau_1 \cdot t)^{1/2}$, where $z(\alpha)$ solves $P\{N(0,1) \leq z(\alpha)\} = 1 - \alpha/2$,

d.) $t(z_2 - z_1)^2 \text{var } v_t \to 0$ (in fact, $(z_2-z_1)^2 \text{var } v_t \sim \alpha^2/t^2$, see GLYNN and IGLEHART (1984)),

e.) β is currently unknown.

Note that β is a reflection of approximation error due to the bias of r_t and skewness/kurtosis effects. It is to be anticipated that the i.i.d.

structure associated with the regenerative viewpoint can be used to reduce these errors. For example, MEKETON and HEIDELBERGER (1982) developed a point estimate which is asymptotically equivalent to r_t, but which significantly reduces bias. Also, GLYNN (1982) proposed a procedure for reducing β in the closely related problem of estimating r on the time scale of regenerative cycles.

As discussed above, the regenerative method is a theoretically sound procedure for the steady state confidence interval problem. The main advantages of the method are:

i.) its good asymptotic properties (for example, $\sigma^2(v_t) = O(1/t^2)$ indicates the accurate "variance constant estimation" possible with the regenerative method),

ii.) the ability to make small-sample corrections to reduce approximation error,

iii.) the i.i.d. structure allows one to develop procedures for a host of other estimation problems (e.g. comparison of stochastic systems; see HEIDELBERGER and IGLEHART (1979)),

iv.) no prior parameters are needed as input for the method, other than run length.

The main disadvantages of the method are:

i.) the requirement to identify regeneration times means that the method is hard to "black box",

ii.) the method may behave unsatisfactorily if the expected time between regenerations is long. (Estimation of parameters for such simulations is likely to be difficult using any method.)

ACKNOWLEDGEMENT

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

ABSTRACT: The regenerative method is a mathematically rigorous method for obtaining confidence intervals for steady state parameters. In this paper the qualitative structure of asymptotic confidence intervals is discussed in general. This structure is then specialized to confidence intervals for steady state parameters produced by the regenerative method. → cont Remarks

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